CONSTRAINTS FROM COSMOGRAPHY IN VARIOUS PARAMETRIZATIONS

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We use cosmography to present constraints on the kinematics of the Universe without postulating any underlying theoretical model *a priori*. To this end, we use a Markov Chain Monte Carlo analysis to perform comparisons to the supernova Ia union 2 compilation, combined with the Hubble Space Telescope measurements of the Hubble constant, and the Hubble parameter datasets. The cosmographic approach to our analysis is revisited and extended for new notions of redshift presented as alternatives to the redshift z. Furthermore, we introduce a new set of fitting parameters describing the kinematical evolution of the Universe in terms of the equation of state of the Universe and derivatives of the total pressure. Our results are consistent with the ΛCDM model, although alternative models, with nearly constant pressure and no cosmological constant, match the results accurately as well.

Keywords: Cosmography; Parametrizations of z; Equation of state.

1. Introduction

In recent years, the wide success of the generally accepted cosmological concordance model has been overshadowed by some inconsistencies, one of which being the problem of dark energy, addressing the unexplained positive accelerated expansion of the Universe.[1] Various efforts, mostly in the form of modifications or extensions of the standard model of cosmology, have been introduced to understand the physical nature of dark energy. Many models have been developed in the literature, but unfortunately none of them has managed to clarify the origin and nature of dark energy satisfactorily.[2] Most of these models are based on the notion of a homogeneous and isotropic Universe, described by the Friedmann-Robertson-Walker (FRW) metric, $ds^2 = -c^2dt^2 + a(t)^2(dr^2/(1-kr^2) + r^2 \sin^2\theta d\phi^2 + r^2 d\theta^2)$. Given the considerable amount of proposals to resolve the issue of dark energy and the difficulties in distinguishing fairly between models and evaluating precisely the degree of accordance between a model and the data, it is desirable to develop an analysis which describes solely the kinematics of the Universe without relying implicitly on a particular model.[3] The purpose of this work is twofold. We first discuss

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the concept of cosmography, a technique of data analysis able to fix bounds on the observable Universe from a model-independent point of view; giving particular regard to developing a viable cosmographic redshift parametrization, which reduces the systematic errors on the fitting coefficients. In addition to that, we derive constraints on the equation of state (EoS) parameter of the Universe directly from data, alleviating the degeneracy problem between cosmological models.

2. The experimental techniques of Cosmography

In this section, we present the basic principles of cosmography and illustrate the way of performing the cosmographic analysis. By involving the cosmological principle only, and correspondingly the FRW metric, it is possible to infer in which way dark energy or alternative components are influencing the cosmological evolution, without implicitly presuming any specific properties or nature of these components. The idea is to expand the most relevant observables such as the Hubble parameter or cosmological distances into power series, and introducing cosmological parameters directly related to these observable quantities.[3] In doing so, it is possible to appraise which models are well in accordance with data and which ones should be discarded as a consequence of not satisfying the basic demands of cosmography. In expanding the luminosity distance d_L into a Taylor series in terms of the cosmological redshift z, we introduce two further notions of redshift, defined as $y_1 = \frac{z}{1+z}$ and $y_4 = \arctan z$. These parameterizations are designed to reduce some disadvantages of the commonly used and well-known notion of the redshift z for the analysis, as e.g. the loss of convergence of the power series for values of $z > 1$. In particular, while y_1 was previously introduced in the literature,[4] we propose to use y_4, which has been obtained by requiring a better convergence behavior of d_L. Our recipe for determining a redshift variable consists in satisfying three considerations: a) the luminosity distance should not behave too steeply in the interval $z < 1$, b) the luminosity distance should not exhibit sudden flexes and c) the curve should be one-to-one invertible. It turns out that the newly introduced y_4 is more suitable for a cosmographic analysis than y_1. For y_4, the parametrization of the luminosity distance is given by $d_L = c/H_0 \cdot \Big[y_4 + y_4^2 \cdot \Big(1/2 - q_0/2\Big) + y_4^3 \cdot \Big(1/6 - j_0/6 + q_0/6 + q_0^2/2\Big) + \mathcal{O}(y_4^4)\Big]$. Furthermore, to counteract the problem of high inaccuracies which is created by cutting the power series expansions too early, we have expanded all quantities up to sixth order. For the numerical fits we made use of the recent data of Union 2 supernovae Ia, of the Hubble Space Telescope (HST) measurements of the Hubble parameter, and of the $H(z)$ compilations,[5] using a Markov Chain Monte Carlo method by modifying the publicly available code CosmoMC.[6] In addition to our generalizations regarding different notions of redshift, we also include a parametrization of the cosmological distance in terms of the EoS parameter ω of the Universe and of the derivatives P_i of the total pressure. This allows us to directly fit the EoS parameter of the Universe from data without having to undergo disadvantageous error propagation in calculating the values from the cosmographic series. This procedure gives clear

constraints on the EoS parameter and on the pressure derivatives in the framework of General Relativity, and thus provides a direct way to compare the predictions of a model for the EoS to observational data.[7] The parametrization of the luminosity distance in terms of the EoS parameter set and as a function of y_4 is given by $d_L(y_4) = c/H_0 \cdot \Big[y_4 + y_4^2/4 \cdot \Big(1-3\omega\Big) + y_4^3 \cdot \Big(5/24 - P_1/4H_0^2 + \omega + 9\omega^2/8\Big) + \mathcal{O}(y_4^4)\Big]$. The numerical results for the parameters of the cosmographic series, i.e. H_0, q_0, j_0 etc., using the newly introduced redshift y_4, can be found in Table 1. The numerical results show a good agreement with ΛCDM, although they seem to be compatible with dark energy possessing constant pressure and an evolving equation of state as well. The corresponding cosmological model[8] appears to be quite indistinguishable from ΛCDM.

Table 1. Table of best fits and their likelihoods (1σ) for redshift y_4, for the three sets of parameters $\mathcal{A} \equiv \{H_0, q_0, j_0, s_0\}$, $\mathcal{B} \equiv \{H_0, q_0, j_0, s_0, l_0\}$ and $\mathcal{C} \equiv \{H_0, q_0, j_0, s_0, l_0, m_0\}$. Set 1 of observations is Union 2 + HST. Set 2 of observations is Union 2 + HST + $H(z)$.

Parameter χ^2_{min}	$\mathcal{A}$, Set 1 530.3	$\mathcal{A}$, Set 2 544.8	$\mathcal{B}$, Set 1 529.7	$\mathcal{B}$, Set 2 544.6	$\mathcal{C}$, Set 1 529.9	$\mathcal{C}$, Set 2 544.5
H_0	$74.55\ ^{+7.54}_{-7.53}$	$73.71\ ^{+5.29}_{-5.24}$	$73.95\ ^{+7.99}_{-7.22}$	$73.43\ ^{+6.05}_{-5.74}$	$74.12\ ^{+8.27}_{-7.78}$	$73.27\ ^{+6.86}_{-5.91}$
q_0	$-0.7492\ ^{+0.5899}_{-0.6228}$	$-0.6504\ ^{+0.4275}_{-0.3303}$	$-0.4611\ ^{+0.5422}_{-0.6710}$	$-0.7230\ ^{+0.5851}_{-0.4585}$	$-0.4842\ ^{+2.7126}_{-0.9280}$	$-0.7284\ ^{+0.6062}_{-0.4838}$
j_0	$2.558\ ^{+7.441}_{-8.913}$	$1.342\ ^{+1.391}_{-1.780}$	$-3.381\ ^{+10.613}_{-2.149}$	$2.017\ ^{+3.149}_{-3.022}$	$-1.940\ ^{+8.041}_{-2.148}$	$2.148\ ^{+3.414}_{-4.036}$
s_0	$9.85\ ^{+74.69}_{-26.69}$	$3.151\ ^{+3.920}_{-1.771}$	$-37.67\ ^{+89.51}_{-60.10}$	$5.278\ ^{+13.076}_{-14.732}$	$-13.48\ ^{+71.65}_{-31.28}$	$2.179\ ^{+42.126}_{-35.919}$
l_0	–	—	$N.C.$	$-0.13\ ^{+96.75}_{-65.87}$	$N.C.$	$-11.60\ ^{+193.88}_{-187.96}$
m_0	–	–	–	–	$N.C.$	$70.9\ ^{+2497.8}_{-2254.5}$

Note: H_0 is given in Km/s/Mpc. *N.C.* means the results are not conclusive - the data do not constrain the parameters sufficiently.

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